We perform microwave spectroscopy of Andreev states in superconducting weak links tailored in an InAs-Al (core-full shell) epitaxially grown nanowire. The spectra present distinctive features with bundles of four lines crossing when the superconducting phase difference across the weak link is 0 or π. We interpret these features as arising from zero-field spin-split Andreev states. A simple analytical model, which takes into account the Rashba spin-orbit interaction in a nanowire containing several transverse subbands, explains these features and their evolution with magnetic field. Our results show that the spin degree of freedom is addressable in Josephson junctions and constitute a first step towards its manipulation. [Full article]

Synopsis: Putting a Spin on the Josephson Effect